State Junior Mathematics Contest
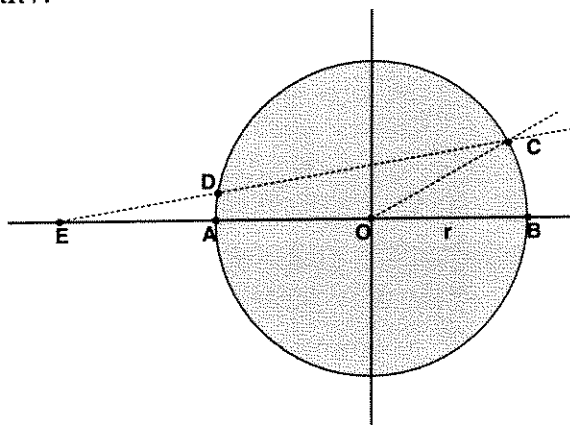## Spring 2008

1. If $a + b = 4$ and $a^2 + b^2 = 9$, then $a^3 + b^3 =$

    (a) 22   (b) 16   (c) 64   (d) 28   (e) 36

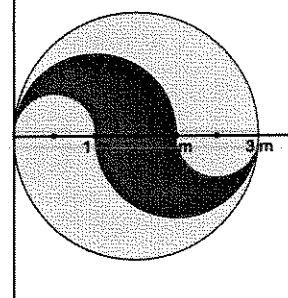2. Points A, B, C lie on a circle with radius r centered at O. Segment DE has length r.

    If $m(< BOC) = 30°$, then $M(< BEC) =$

    (a) $15°$
    (b) $12°$
    (c) $10°$
    (d) $20°$
    (e)   cannot be determined without more infomation

3. The circle to the right has diameter $3\,m$. Find the area of the shaded region if its boundary consists of semicircles:
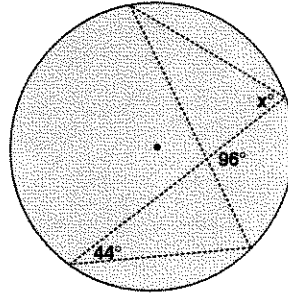

    (a) $3\pi\, m^2$ (b) $\pi\, m^2$ (c) $\frac{3}{2}\pi\, m^2$ (d) $\frac{3}{4}\pi\, m^2$
    (e) $\frac{3}{8}\pi\, m^2$

4. If f is a function such that $f(x - 1) = x^2 - 3x + 5$ then $f(x + 1) = ?$

    (a) $x^2 + x + 3$
    (b) $x^2 - x + 3$
    (c) $x^2 + x$
    (d) $x^2 - 3x + 7$
    (e)   none of these

5. Chords are drawn in a circle as shown. The value of x is

(a) 44   (b) 48   (c) 52   (d) 34   (e) 84

6. A 3×3 magic square uses integers $1, 2, \ldots, 9$ once each in such a way that each column, each row, and each diagonal sums to 15. Find the value of N for the magic square, a portion of which is shown below:

8		
N		7

(a) 1   (b) 2   (c) 3   (d) 4   (e) 5

7. If $i = \sqrt{-1}$, then $\dfrac{8 - 4i}{4 + 2i} =$

(a) $2 - \dfrac{8}{3}i$ (b) 2   (c) $2 - 2i$ (d) $\dfrac{6}{5} - \dfrac{8}{5}i$ (e) $\dfrac{10}{3} - \dfrac{8}{3}i$

8. The ratio of the circumference of a circle to the perimeter of an inscribed square is:

(a) $\dfrac{\pi\sqrt{2}}{3}$ (b) $\dfrac{\pi\sqrt{2}}{4}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{3}$ (e) none of these

9. Which of the following conditions imply that the real number x is rational?

    I. $\sqrt{x}$ is rational

    II. x^2 and x^3 are rational

    III. x^2 and x^4 are rational

        (a)   I only
        (b)   I and II only
        (c)   I and III only
        (d)   II and III only
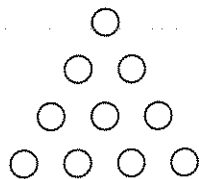   (e)   I, II, and III

10. If the roots of $x^2 + bx + c = 0$ are π and $\sqrt{2}$, then $b =$

        (a) $\pi\sqrt{2}$
        (b) $2\pi\sqrt{2}$
        (c) $2(\pi + \sqrt{2})$
        (d) $\pi + \sqrt{2}$
        (e) $-(\pi + \sqrt{2})$

11. One year, there were exactly four Fridays and four Mondays in July. What day of the week was July 20?

        (a)   Sunday
        (b)   Monday
        (c)   Wednesday
        (d)   Thursday
        (e)   Saturday

12. What is the smallest number of circles that must be removed from the figure so that no three remaining circles form an equilateral triangle?

$$\circ$$
$$\circ \quad \circ$$
$$\circ \quad \circ \quad \circ$$
$$\circ \quad \circ \quad \circ \quad \circ$$

    (a)   1   (b)   2   (c)   3   (d)   4   (e)   5

13. How many four-digit numbers are there with all odd digits?

    (a)   625   (b)   5!   (c)   5000   (d)   5001   (e)   20

14. Which of the following numbers is a factor of 68,574,961?

   (a) 3   (b) 9   (c) 7   (d) 2   (e) 11

15. An equilateral triangle is divided into more than one smaller equilateral triangles. What is the smallest possible number of such triangles?

   (a) 3   (b) 4   (c) 5   (d) 6   (e) 9

16. Suppose 50 cities are connected by roads in such a way that three roads lead in and out of each city. How many roads are there total?

   (a) 50   (b) 75   (c) 100   (d) 125   (e) 150

17. Solve $\log(5x) + \log(x - 1) = 2$.

   (a) 3   (b) 5   (c) -4 (d) 3 and 5   (e) 5 and -4

18. How many three-digit whole numbers have the property that doubling them results in reversing their digits? (For instance 125 does not have this property since $2(125) = 250$ which does not equal 521,the number obtained by reversing the digits of 125. Also, 025 is not considered a three-digit number, but rather the two-digit number 25.)

   (a) 0   (b) 1   (c) 2   (d) 6   (e) none of the above

19. A boy and a girl are sitting on the porch. "I'm a boy," says the child with black hair. "I'm a girl," says the child with red hair. At least one of them is lying. What is the maximum number of statements below that can be true?

   (I) The person with red hair is a boy.
   (II) The person with red hair is a girl.
   (III) The person with black hair is a girl.

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e)   There is not enough information to determine the answer.

20. Mary had a coin purse with fifty coins (which are either pennies, nickles, dimes or quarters) totaling exactly $1.00. Unfortunately, while counting her change, she dropped one coin. What is the probability that it was a penny?

  (a) 50%
  (b) 75%
  (c) 85%
  (d) 90%
  (e) There is not enough information to determine the answer.

21. Given | S | T | A | T | E | | M | A | T | H | how many arrangements are there of these blocks?

  (a) 10!   (b) $\dfrac{10!}{5!}$ (c) $\dfrac{10!}{3!}$ (d) $\dfrac{10!}{12}$ (e) $\dfrac{10!}{6}$

22. In triangle ABC, the measure of the angle at vertex A is three times the measure of the angle at vertex B and half the measure of the angle at vertex C. What is the measure of the angle at vertex A?

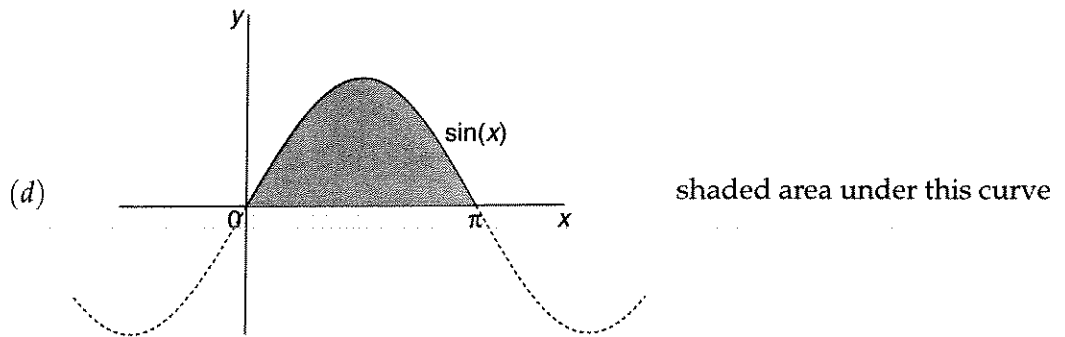 (a) 30° (b) 36° (c) 54° (d) 60° (e) 72°
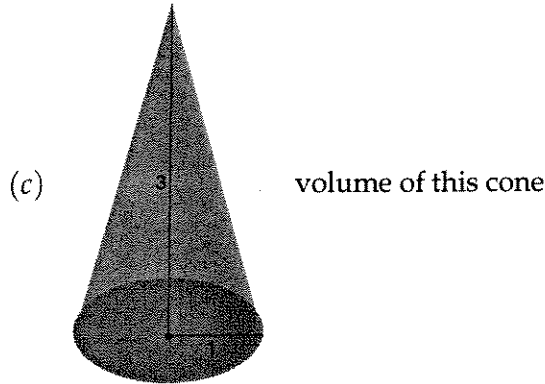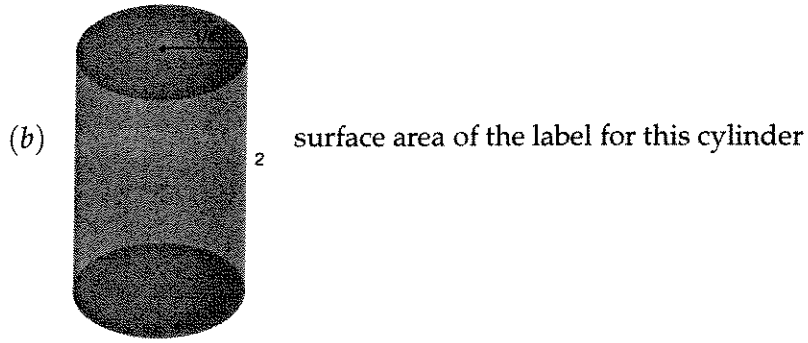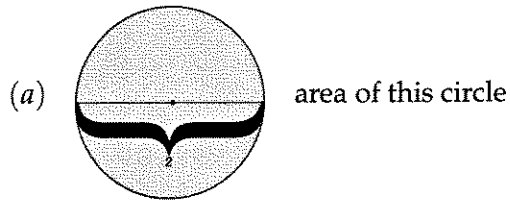
23. A park has the shape of a regular hexagon of sides 2 km each. Allice walks a distance of 5km around the perimeter. What is the direct distance between the start point and the end point?

  (a) $\sqrt{13}$ (b) $\sqrt{14}$ (c) $\sqrt{15}$ (d) $\sqrt{16}$ (e) $\sqrt{17}$

24. A nanobillion is

  (a) .01   (b) 0.1   (c) 1.0   (d) 10   (e) 100

25. Which of the following is **not** equal to π?

(a)   area of this circle

(b)   surface area of the label for this cylinder

(c)   volume of this cone

(d)   shaded area under this curve

$\sin(x)$

(e)   all of the above are numerically equal to π

26. I have three cakes, each divided into 5 equal pieces. A serving is $\frac{2}{5}$ of a cake. How many servings do I have altogether?

   (a) $\frac{2}{15}$ (b) $1\frac{1}{5}$ (c) $3\frac{2}{5}$ (d) $7\frac{1}{5}$ (e) $7\frac{1}{2}$

27. Given an equilateral triangular piece of cardboard, create an open box (i.e., without a lid) by cutting the same shape from each corner and folding up the flaps. What is the height of the box of maximal volume? (Assume length of the leg of original cardboard piece is x.)

(a) $\frac{x}{6}$ (b) $\frac{\sqrt{3}x}{18}$ (c) $\frac{x}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{9}x$ (e) $\frac{1}{3}x$

28. Amaliea is putting her stack of pennies into rolls, keeping out the shiny ones. She notices that every other penny she picks up is dull and every third one is discolored and every fourth one is nicked or bent. How many pennies will she have to roll up if she ends up with fifty shiny pennies?

(a) 50 (b) 100 (c) 120 (d) 150 (e) 160

29. In this expression $ax + by + c = d$, which constants and coefficients determine the y-intercept?

(a) a, b and c
(b) b, c and d
(c) a, b and d
(d) a, c and d
(e) a, b,c and d

30. Solve for x.
$$8^{2x} = 2^x \left(\frac{64^6}{2} \right)$$

(a) 5 (b) 6 (c) 7 (d) 10 (e) 11