

**State Senior Mathematics Contest
Spring 2007**

1. What is the greatest divisor of $19!$ and $19! + 17$?

- (a) 1 (b) 17 (c) 19 (d) $19!$ (e) $17!$

$$\begin{aligned} 19! &= 17(19(18)16!) \\ 19! + 17 &= 17(19(18)16! + 1) \end{aligned}$$

2. The decimal $0.\overline{9} = 0.999\dots$ is equal to

- (a) 1 (b) $1 - (\frac{9}{10})^{10}$ (c) $(\frac{9}{10})^{\frac{10}{9}}$ (d) $999/1000$ (e) $9/10$

$$\text{Let } n = 0.\overline{9} \quad \text{then } 10n = 9.\overline{9}$$

$$\begin{aligned} \Rightarrow \quad 10n &= 9.\overline{9} \\ - \quad n &= 0.\overline{9} \\ \hline 9n &= 9 \\ n &= 1 \end{aligned}$$

3. If you lose 20% on an investment during the first year and gain 25% the following year, what is your net gain over the two years?

- (a) 0% (b) 5% (c) 2.5% (d) -5% (e) 1.25%

Let $x =$ original investment
after 1st year, investment = $0.8x$
after 2nd year, investment = $1.25(0.8x)$
 $= \frac{5}{4} \left(\frac{4}{5}x \right) = 1x$
 \Rightarrow it is back to original value \Rightarrow 0 net gain

4. How many divisors does the number 2007 have?

- (a) 2 (b) 3 (c) 4 (d) 6 (e) 8

2007
 \wedge
 9 223
 \wedge prime
 3 3

divisors
1, 3, 9, 223, 669, 2007

5. The number 2^{29} is a 9-digit number with distinct digits. Which digit is missing?

- (a) 0 (b) 3 (c) 4 (d) 5 (e) 7

$$2^{29} = 2^{10} 2^{10} 2^9 = 1024(1024)(512)$$

$\begin{array}{r} 1024 \\ \times 1024 \\ \hline 4096 \\ 2048 \\ 10240 \\ \hline 1048576 \end{array}$	$\begin{array}{r} 1048576 \\ \times 512 \\ \hline 2097152 \\ 1048576 \\ 5242880 \\ \hline 536870912 \end{array}$
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$\Rightarrow 4$ is missing

6. If this pattern continues, where would the number 289 appear?

		1		
		3	5	
	7	9	11	
13	15	17	19	

- (a) 8th element in row 16
 (b) 9th element in row 17
 (c) 9th element in row 18
 (d) last element in row 17
 (e) last element in row 18

			1						
			3	5					
		7	9	11					
	13	15	17	19					
	21	23	25	27	29				
31	33	35	37	39	41				
43	45	47	49	51	53	55			

perfect squares

$289 = 17^2 \Rightarrow 17^{\text{th}}$ row, in middle

$\Rightarrow 9^{\text{th}}$ element

7. Consider an infinite geometric series with first term a and common ratio r . If the sum is 4 and the second term is $\frac{3}{4}$, then a possible choice of a and r is

(a) $a = \frac{7}{4}, r = \frac{3}{7}$

(b) $a = 2, r = \frac{3}{4}$

(c) $a = \frac{3}{2}, r = \frac{1}{2}$

(d) $a = 3, r = \frac{1}{4}$

(e) $a = 1, r = \frac{1}{4}$

$a + ar + ar^2 + ar^3 + \dots$

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad |r| < 1$$

$$\frac{a}{1-r} = 4 \quad \text{and} \quad ar = \frac{3}{4}$$

$$\Rightarrow \frac{3/4r}{1-r} = 4$$

$$\frac{3}{4r} = 4 - 4r \Rightarrow 3 = 16r - 16r^2$$

$$\Rightarrow 16r^2 - 16r + 3 = 0$$

$$(4r-1)(4r-3) = 0$$

8. For all $x \in (0, 1)$, which statement is true?

(a) $e^x < 1+x$

(b) $\ln(1+x) < x$

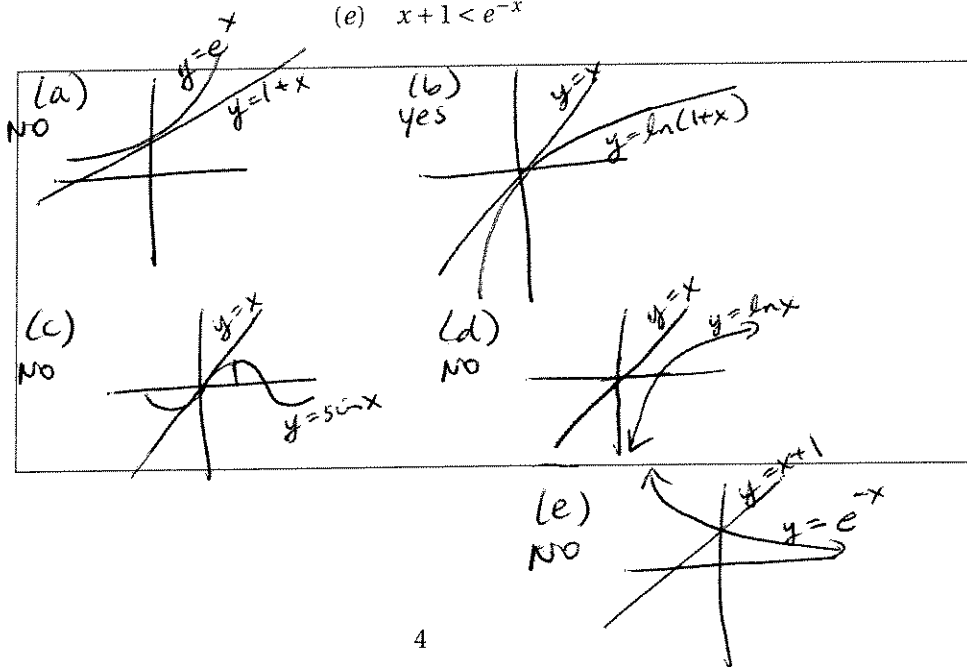
(c) $x < \sin x$

(d) $x < \ln x$

(e) $x+1 < e^{-x}$

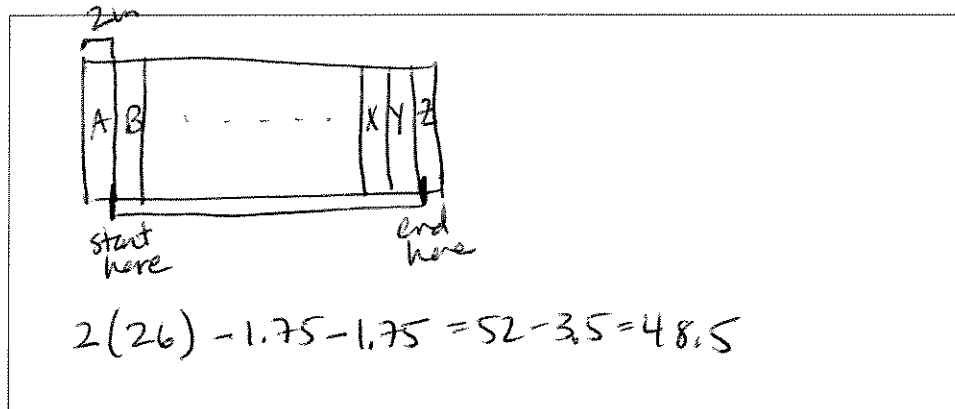
$r = 1/4 \quad r = 3/4$

$a = 3 \quad a = 1$



9. A set of 26 encyclopedias (one for each letter) is placed on a bookshelf in alphabetical order from left to right. Each encyclopedia is 2 inches thick including the front and back covers. Each cover (front or back) is $\frac{1}{4}$ inch thick. A bookworm eats straight through the encyclopedias, beginning inside the front cover of volume A and ending after eating through the back cover of volume z. How many inches of book did the bookworm eat?

- (a) 48 (b) 48.5 (c) 51.25 (d) 51.5 (e) 51.75



10. What is the smallest positive integer so that $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^n = 1$

- (a) 0 (b) 2 (c) 4 (d) 8 (e) 16

We know $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$
 $= \frac{1}{2} + i - \frac{1}{2} = i$
 and $i^4 = 1$
 $\Rightarrow \left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2\right]^4 = 1$
 or $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^8 = 1$

11. One hundred balls labelled 1 through 100 are placed in a bag. Four balls are removed from the bag, one by one. What is the probability that the label on the first ball is higher than the label on the last?

(a) $5/4$ (b) $1/2$ (c) 0 (d) $49/50$ (e) $4/5$

Due to symmetry, it's equally likely that 1st ball > last ball as last ball > 1st ball
 $\Rightarrow P = 1/2$.

OR $P = \frac{\frac{99(100)}{2}(98)(97)}{{}_{100}P_4} = \frac{100(99)(98)(97)}{2 \left(\frac{100!}{96!} \right)} = \frac{1}{2}$

Since ${}_{100}P_4 =$ total # of 4-ball draws
 + there are $\frac{99(100)}{2}(98)(97)$ ways to have 1st ball higher than last ball

12. What are the dimensions of the rectangle with the largest area that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$?

(a) 2×3
 (b) $2\sqrt{3} \times 3\sqrt{3}$
 (c) $\sqrt{3} \times \frac{3}{2}\sqrt{3}$
 (d) $2\sqrt{2} \times 3\sqrt{2}$
 (e) $\sqrt{2} \times \frac{3}{2}\sqrt{2}$

I'll maximize $1/4$ of rectangle.
 maximize $A = xy$ given $\frac{x^2}{4} + \frac{y^2}{9} = 1$
 $A = x \left(\frac{3}{2} \sqrt{4-x^2} \right)$
 $0 < x < 2$
 $= \frac{3x}{2} \sqrt{4-x^2}$
 $\frac{dA}{dx} = \frac{3}{2} \sqrt{4-x^2} - \frac{3x^2}{2\sqrt{4-x^2}} = 0$
 $y^2 = 9 - \frac{9}{4}x^2$
 $y = \sqrt{9 - \frac{9}{4}x^2}$
 $y = \frac{3}{2} \sqrt{4-x^2}$

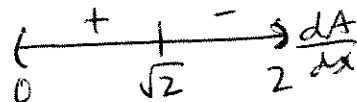
$$3(4-x^2) - 3x^2 = 0$$

$$12 - 6x^2 = 0$$

$$2 = x^2$$

$$\sqrt{2} = x$$

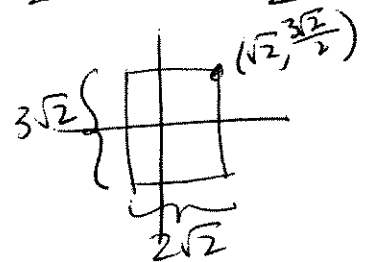
(since $x > 0$)



max \nearrow

$$\Rightarrow x = \sqrt{2} \text{ makes max } A$$

$$\Rightarrow y = \frac{3}{2} \sqrt{4-2} = \frac{3\sqrt{2}}{2}$$



13. If you place these expressions in increasing order, which one will be in the middle?

$$(a) \sum_{k=1}^{1000} (-1)^k = \underbrace{(-1+1)+(-1+1)+\dots+(-1+1)}_{500 \text{ pairs}} = 0$$

$$(b) \sum_{k=2}^{20} k^2 = \frac{20(20+1)(20(2)+1)}{6} = \frac{20(21)(41)}{6} = 70(41) = 2870$$

Harmonic series diverges

$$(c) \sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \rightarrow \infty$$

$$(d) \sum_{k=1}^{100} k = \frac{100(100+1)}{2} = 50(101) = 5050$$

$$(e) \sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k = 2\left[\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - 1\right] = 2\left[\frac{1}{1-\frac{1}{2}} - 1\right] = 2$$

(a) 0

(e) 2

(b) 2870

(d) 5050

(c) ∞

14. The diagonals of a rhombus are 12 and 24. Determine the radius of the circle inscribed in the rhombus.

- (a) $6\sqrt{5}$
 (b) $12\sqrt{5}$
 (c) $\frac{6}{\sqrt{5}}$
 (d) $\frac{12}{\sqrt{5}}$
 (e) Cannot inscribe a circle in a rhombus

Diagonals of a rhombus are \perp to each other.

We have

$\frac{6\sqrt{5}}{12} = \frac{6}{r}$
 $r(6\sqrt{5}) = 72$
 $r = \frac{12}{\sqrt{5}}$

$6^2 + 12^2 = 180 = d^2$
 $\Rightarrow d = 6\sqrt{5}$

} similar Δ s

15. If w, x, y, z are positive real numbers such that $w + x + y + z = 2$, then

$$N = (w + x)(y + z)$$

satisfies

- (a) $0 \leq N \leq 1$
 (b) $1 \leq N \leq 2$
 (c) $2 \leq N \leq 3$
 (d) $3 \leq N \leq 4$
 (e) $4 \leq N \leq 5$

Try easy case $x = y = z = w = \frac{1}{2}$
 $\Rightarrow N = (\frac{1}{2} + \frac{1}{2})(\frac{1}{2} + \frac{1}{2}) = 1 \Rightarrow$ either (a) or (b)

Try $w = \frac{3}{2}$ and $x = y = z = \frac{1}{6}$
 $\Rightarrow N = (\frac{3}{2} + \frac{1}{6})(\frac{1}{6} + \frac{1}{6}) = \frac{10}{6}(\frac{2}{6}) = \frac{5}{3}(\frac{1}{3}) = \frac{5}{9} < 1$
 \Rightarrow (a)

16. As $x \rightarrow \infty$, the function $\left(\frac{x-3}{x+2}\right)^x$ approaches

- (a) e (b) $\frac{1}{e}$ (c) e^{-5} (d) e^5 (e) 1

★ You can also use L'Hopital's Rule here, but it is longer.

as $x \rightarrow \infty$
 $(x+2) \rightarrow \infty$
 also

Remember defn of $e^r = \lim_{x \rightarrow \infty} \left(1 + \frac{r}{x}\right)^x$

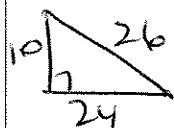
$$\left(\frac{x-3}{x+2}\right)^x = \left(\frac{x+2-5}{x+2}\right)^x = \left(1 + \frac{-5}{x+2}\right)^x = \frac{\left(1 + \frac{-5}{x+2}\right)^{x+2}}{\left(1 + \frac{-5}{x+2}\right)^2}$$

$$\Rightarrow \lim_{(x+2) \rightarrow \infty} \frac{\left(1 + \frac{-5}{x+2}\right)^{x+2}}{\left(1 + \frac{-5}{x+2}\right)^2} = \frac{\lim_{(x+2) \rightarrow \infty} \left(1 + \frac{-5}{x+2}\right)^{x+2}}{\lim_{(x+2) \rightarrow \infty} \left(1 + \frac{-5}{x+2}\right)^2} = \frac{e^{-5}}{1^2} = e^{-5}$$

17. Triangle ABC has sides 10, 24, and 26 cm long. A rectangle that has an area equal to that of the triangle has width 3 cm. Find the perimeter of the rectangle.

- (a) 40 cm (b) 43 cm (c) 56 cm (d) 68 cm (e) 86 cm

We know this is a right Δ because the side lengths are multiple of the 5-12-13 Pythagorean triple.



$$A = \frac{1}{2}(24)(10) = 120$$



$$A = 3x$$

$$\Rightarrow 120 = 3x$$

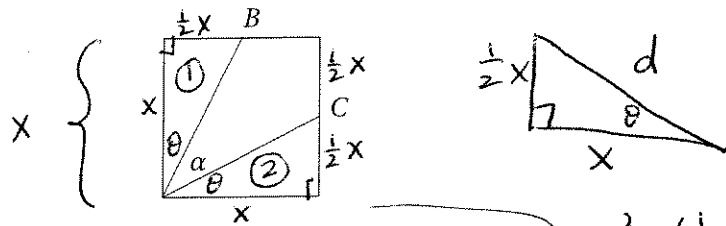
$$40 = x$$

$$\Rightarrow \text{perimeter} = 2(3) + 2(x) = 6 + 2(40)$$

$$= 6 + 80$$

$$= 86$$

18. Given the square with midpoints B and C . What is the $\sin \alpha$?



- (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{2}{\sqrt{5}}$

$$x^2 + \left(\frac{1}{2}x\right)^2 = d^2$$

$$d = \frac{\sqrt{5}}{2}x$$

$\Delta 1 \cong \Delta 2$ by SSS

$$2\theta + \alpha = 90^\circ$$

$$\alpha = 90^\circ - 2\theta = 90^\circ - 2\theta$$

$$\sin \alpha = \sin(90^\circ - 2\theta) = \cos(2\theta)$$

$$= \cos^2 \theta - \sin^2 \theta = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

$$\cos \theta = \frac{x}{\frac{\sqrt{5}}{2}x} = \frac{2}{\sqrt{5}}$$

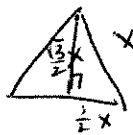
$$\sin \theta = \frac{\frac{1}{2}x}{\frac{\sqrt{5}}{2}x} = \frac{1}{\sqrt{5}}$$

19. If the area of a circle is equal to the area of an equilateral triangle, then the ratio of the side of the triangle to the radius of the circle is closest to which number?

- (a) 3 (b) 4 (c) 5 (d) 6 (e) 7



$$A_o = \pi r^2$$



$$A_{\Delta} = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right)$$

$$= \frac{\sqrt{3}}{4}x^2$$

$$A_o = A_{\Delta} \Leftrightarrow \pi r^2 = \frac{\sqrt{3}}{4}x^2 \Leftrightarrow \frac{x^2}{r^2} = \frac{4\pi}{\sqrt{3}}$$

$$\frac{x}{r} = \sqrt{\frac{4\pi}{\sqrt{3}}} \approx 2\sqrt{\frac{\pi}{\sqrt{3}}}$$

10

$$\frac{\pi}{\sqrt{3}} \sim 1.8 \Rightarrow 2\sqrt{1.8} \sim 2(1.35)$$

(since $\sqrt{1.8} < \sqrt{2}$
 $\times 1.4$) $\approx 2.7 \approx 3$

20. If this multiplication problem works in base b , what is b ?

$$(15_b)(15_b) = 321_b$$

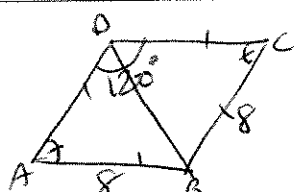
- (a) 4 (b) 6 (c) 7 (d) 8 (e) 9

Just look at last digits. We know $5_b \times 5_b = \text{something that ends in } 1$ which means the base must be one more than a group of 5's.
 \Rightarrow Try base 6.

$$\begin{array}{r} 15_6 \\ \times 15_6 \\ \hline 131 \\ 15 \\ \hline 321_6 \end{array} \checkmark$$

21. A rhombus with sides of 8 cm and an angle of 120° will have an area closest to.

- (a) 35 cm^2 (b) 45 cm^2 (c) 55 cm^2 (d) 60 cm^2 (e) 65 cm^2



$\triangle ABD \cong \triangle CBD$ by SAS
 $m\angle A = 120^\circ \Rightarrow m\angle ABD = 60^\circ$

Area of rhombus
 $= 2(16\sqrt{3})$
 $= 32\sqrt{3}$
 $\approx 32(1.7)$
 $= 54.4$

$A = \frac{\sqrt{3}}{4} s^2 = \frac{\sqrt{3}}{4} (8^2)$
 Area = $16\sqrt{3}$

22. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has

- (a) both roots in $[a, b]$
- (b) both roots in $(-\infty, a)$
- (c) both roots in (b, ∞)
- (d)** one root in $(-\infty, a)$ and the other in (b, ∞)
- (e) one root in $[a, b]$ and the other in (b, ∞)

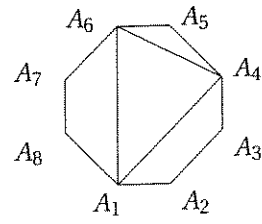
$$x^2 + (-a-b)x + (ab-1) = 0$$

$$x = \frac{(a+b) \pm \sqrt{(a+b)^2 - 4(ab-1)}}{2} = \frac{a+b}{2} \pm \frac{\sqrt{a^2 - 2ab + b^2 + 4}}{2}$$

$$x = \frac{a+b}{2} \pm \frac{\sqrt{(a-b)^2 + 4}}{2}$$

\uparrow midpt of $[a, b]$ \pm some stuff \Rightarrow interval is symmetric wrt $\frac{a+b}{2}$

23. How many different triangles can you draw as in the figure, if the three vertices have to be among the shown points A_1, \dots, A_8 ? \Rightarrow either (a) or (d)



but $\frac{\sqrt{(a-b)^2 + 4}}{2} > \frac{b-a}{2}$
 \Rightarrow answers are outside $[a, b]$
 \Rightarrow (d)

- (a) $8(7)(6)$
- (b)** 56
- (c) $8!$
- (d) $3!$
- (e) 24

8 vertices, choose 3 to form a Δ

$${}^8C_3 = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2}$$

$$= 8 \cdot 7 = 56$$

24. What is the value of

$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$$

- (a) 1 (b) $\frac{1}{2}$ (c) $1 + \sqrt{2}$ (d) $-1 \pm \sqrt{2}$ (e) $-1 + \sqrt{2}$

Let $x = \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$ \Rightarrow $x = \frac{1}{2 + x}$

$$x(2+x) = 1$$

$$x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$$

but $x > 0 \Rightarrow x = -1 + \sqrt{2}$

25. Paul and Judy play the exciting game "throw a coin six times". If the coin shows heads, Paul gets a point, if tails, Judy gets a point. After six throws, they compare their scores. How likely is it that the game will be a tie?

- (a) $\frac{1}{2}$ (b) $\frac{5}{16}$ (c) $\frac{1}{4}$ (d) $\frac{7}{16}$ (e) $\frac{3}{16}$

			1	1			
		1	2	1			
		1	3	3	1		
		1	4	6	4	1	
		1	5	10	10	5	1
	1	6	15	20	15	6	1
0H	1H	2H	3H	4H	5H	6H	

2^6 possibilities w/ 6-coin throw

For a tie, there would be 3H and 3T.

$$P(3H) = \frac{20}{2^6} = \frac{5}{2^4}$$

$$= \frac{5}{16}$$

26. If $f(\sin(x)) = \sin(3x)$, then $f(\cos(30^\circ)) = ?$

- (a) 0 (b) 1 (c) -1 (d) $\sqrt{\frac{3}{2}}$ (e) $\frac{1}{2}$

$$\begin{aligned} \cos 30^\circ &= \sin 60^\circ \\ \Rightarrow f(\cos(30^\circ)) &= f(\sin(60^\circ)) \\ &= \sin(3(60^\circ)) \\ &= \sin 180^\circ = 0 \end{aligned}$$

27. If the equation $(\frac{1}{4})^x + (\frac{1}{2})^{x-1} + b = 0$ has a positive solution, then the real number b is in what interval?

- (a) $-\infty < b < 1$
 (b) $-\infty < b < -2$
 (c) $-\infty < b < 0$
 (d) $-3 < b < 0$
 (e) $-\infty < b < -3$

$$\begin{aligned} & \left[\left(\frac{1}{2} \right)^x \right]^2 + 2 \left[\left(\frac{1}{2} \right)^x \right] + b = 0 \quad \text{like a quadratic eqn} \\ \left(\frac{1}{2} \right)^x &= \frac{-2 \pm \sqrt{4-4b}}{2} = -1 \pm \sqrt{1-b} \quad \left(\begin{array}{l} \text{only consider} \\ \text{positive solution} \end{array} \right) \\ \left(\frac{1}{2} \right)^x &= -1 + \sqrt{1-b} \\ \ln \left(\frac{1}{2} \right)^x &= \ln(\sqrt{1-b} - 1) \\ x &= \frac{\ln(\sqrt{1-b} - 1)}{-\ln 2} \Rightarrow \text{We need } (\ln(\sqrt{1-b} - 1)) < 0 \end{aligned}$$

∴ we know $x > 0$ we can only take \ln of positive #'s

$$\Rightarrow 0 < \sqrt{1-b} - 1 < 0$$

$$1 < \sqrt{1-b} < 2$$

$$1 < 1-b < 4$$

$$0 < -b < 3$$

$$0 > b > -3$$

$$\Leftrightarrow -3 < b < 0$$

28. If $f(x) = 3x^2 - x + 4$, $f(g(x)) = 3x^4 + 18x^3 + 50x^2 + 69x + 48$, then what is one of the sums of all the coefficients of $g(x)$?

- (a) 8 (b) 1 (c) 3 (d) 7 (e) 0

$g(x)$ must be quadratic polynomial
 $\rightarrow g(x) = ax^2 + bx + c$
 $f(g(x)) = 3(ax^2 + bx + c)^2 - (ax^2 + bx + c) + 4$
 $= (3a^2)x^4 + (6ab)x^3 + (6ac + 3b^2 - a)x^2 + (6bc - b)x + (3c^2 - c + 4) = 3x^4 + 18x^3 + 50x^2 + 69x + 48$
 $\Rightarrow 3a^2 = 3 \quad 6ab = 18 \quad 6ac + 3b^2 - a = 50$
 $\Leftrightarrow a = \pm 1 \quad b = \frac{18}{6a} = \pm 3 \quad \begin{matrix} a=1 & b=3 & c=4 \\ a=-1 & b=-3 & c=-1/3 \end{matrix}$

a	1	-1
b	3	-3
c	4	-1/3

$\Rightarrow a+b+c = 1+3+4=8$

or $-1-3-1/3 = -23/3$

29. Evaluate $\int_1^3 \frac{x^3 + x^2 + 1}{x^2 + x} dx$

- (a) $4 + \ln \frac{2}{3}$
 (b) $4 - \ln \frac{2}{3}$
 (c) $\frac{9}{2} + \ln \frac{3}{4}$
 (d) $\frac{9}{2} - \ln \frac{4}{3}$
 (e) $\frac{9}{2} - \ln \frac{2}{3}$

(all other coefficients equate w/ these answers)

① do long division

$$\begin{array}{r} x + \frac{1}{x^2+x} \\ x^2+x \overline{) x^3+x^2+1} \\ \underline{-(x^3+x^2)} \quad \quad \quad \\ \quad \quad \quad 1 \end{array}$$

② partial fractions

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$1 = Ax + A + Bx$$

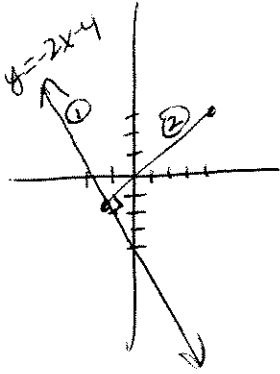
$$A=1 \quad A+B=0 \quad B=-1$$

$$\begin{aligned} \int_1^3 x + \frac{1}{x^2+x} dx &= \frac{x^2}{2} \Big|_1^3 + \int_1^3 \frac{1}{x(x+1)} dx \\ &= \left(\frac{9}{2} - \frac{1}{2}\right) + \int_1^3 \frac{1}{x(x+1)} dx \\ &= 4 + \int_1^3 \left(\frac{1}{x} - \frac{1}{x+1}\right) dx \\ &= 4 + (\ln|x| - \ln|x+1|) \Big|_1^3 \\ &= 4 + (\ln 3 - \ln 4) - (\ln 1 - \ln 2) \\ &= 4 + \ln 3 - \ln 4 + \ln 2 \\ &= 4 + \ln\left(\frac{6}{4}\right) = 4 + \ln\left(\frac{3}{2}\right) \\ &\text{or } 4 - \ln\left(\frac{2}{3}\right) \end{aligned}$$

30. Find the perpendicular distance of the point (4,3) from the line

line ① $y = -2x - 4$.

- (a) $\sqrt{65}$ (b) 9 (c) $2\sqrt{5}$ (d) $3\sqrt{5}$ (e) 4



line ② \perp to ① $m = \frac{1}{2}$ thru $(4, 3)$

$$y - 3 = \frac{1}{2}(x - 4)$$

$$y = \frac{1}{2}x + 1$$

need intersect pt between ① + ②

$$\frac{1}{2}x + 1 = -2x - 4$$

$$x = -2$$

$$\Rightarrow y = 0$$

distance from $(-2, 0)$ to $(4, 3)$

$$d = \sqrt{(0-3)^2 + (-2-4)^2} = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$$