Solutions for Introduction to Polynomial Calculus

Section 1 Problems

Bob Palais

The point-slope form of the equation of a line says that the rise over the run between an arbitrary point on a line (x, y) and a particular point (x_0, y_0) on that line is constant, m, called the slope of the line. This describes a relationship of direct proportionality or linearity between the rise and the run. The rise is the change in y, $y - y_0$, and the run is the change in x, $x - x_0$, so $\frac{y-y_0}{x-x_0} = m$. Since the ratio is undefined for the point (x_0, y_0), it is common to cross multiply so that this point fits the equation explicitly: $y - y_0 = m(x - x_0)$. If you are given two points on a line, they may be used to compute its slope, and either may be used in the point-slope form.

So for (1)-(6) I'm giving not only the slope which the problem asks for but also the point-slope equation of the line.

(1) $m = \frac{2-1}{1-0} = 1$ and the equation is $y - 1 = 1(x - 0)$ or $y - 2 = 1(x - 1)$.

(2) $m = \frac{7-3}{4-2} = 2$ and the equation is $y - 3 = 2(x - 2)$ or $y - 7 = 2(x - 4)$.

(3) $m = \frac{2-1}{3-1} = \frac{1}{2}$ and the equation is $y - 1 = \frac{1}{2}(x - 1)$ or $y - 2 = \frac{1}{2}(x - 3)$.

(4) $m = \frac{2-4}{3-1} = -1$ and the equation is $y - 4 = -1(x - 1)$ or $y - 2 = -1(x - 3)$.

(5) $m = \frac{1-3}{3-(-2)} = -\frac{2}{5}$ and the equation is $y - 3 = -\frac{2}{5}(x - (-2))$ or $y - 1 = -\frac{2}{5}(x - 3)$.

(6) $m = \frac{2-0}{0-(-2)} = 1$ and the equation is $y - 0 = 1(x - (-2))$ or $y - 2 = 1(x - 0)$.

(7) $y - 0 = 2(x - 0)$

(8) $y - 2 = 5(x - 1)$

(9) $y - (-1) = -3(x - 2)$

(10) $y - 1 = \frac{1}{2}(x - 1)$

(11) $y - 5 = -\frac{2}{3}(x - 0)$

(12) $y - 0 = 7(x - (-2))$

I intentionally prefer the $(x - (-a))$ form to $(x + a)$ because it displays the important information more clearly. I do not require or encourage oversimplification of answers! Conversion to slope-intercept form is not required or encouraged either as long as you know how to do it. Usually points other than $x = 0$ are more important and it is better to refer equations to the point of interest. The slope-intercept form is nice when you wish to extend to polynomials in standard form: $a_0 + a_1 x + \ldots + a_n x^n$, but even polynomials have useful forms adapted to another point: $a_0 + a_1(x - c) + \ldots + a_n(x - c)^n$, or even useful 'multiple center' forms: $a_0 + (x - c_1)(a_1 + \ldots + (x - c_n)(a_n]$.

(13) $y = 3x + 1$

(14) $y = \frac{4}{3}x + 2$

(15) Put the equation in slope-intercept form by adding $2y$ to both sides, subtracting 4 from both sides, and dividing by 2: $y = 3x - 2$, so the slope is 3 and the $y-$intercept is -2.

(16) Put the equation in slope-intercept form by subtracting $2x$ from both sides, and dividing by 5: $y = -\frac{2}{5}x + \frac{3}{5}$, so the slope is $-\frac{2}{5}$ and the $y-$intercept is $\frac{3}{5}$.

(17) Parallel lines have the same slope, so $y - 1 = 3(x - 1)$

(18) The equation of any non-vertical line containing the point $(2, -1)$ is $y - (-1) = m(x - 2)$. Parallel lines have the same slope, so $m = \frac{2-0}{3-2} = 2$. So the equation is $y - (-1) = 2(x - 2)$.

(19) The slope of any line perpendicular to a line with slope $m \neq 0$ is $-\frac{1}{m}$, the 'negative reciprocal' rule. So $y - 0 = -\frac{1}{3}(x - 1)$.

(20) To find the midpoint of two points and the bisector of the segment joining them, compute the simple average their horizontal and vertical coordinates respectively: $\frac{0+2}{2} = 1$ and $\frac{0+4}{2} = 2$ so the line goes through the point $(1, 2)$. The slope of the segment is $\frac{4-0}{2-0} = 2$, so the slope of any line perpendicular to it is $-\frac{1}{2}$ and the equation of the line with this slope through that point is $y - 2 = -\frac{1}{2}(x - 1)$.

(21) The slope of any line perpendicular to a vertical line $x = c$ is $m = 0$. So $y - 1 = 0$ or $y = 1$ whose graph is horizontal.

(22) The equation of any line perpendicular to a horizontal line $y = c$ is of the form $x = c$ and its slope is undefined. So $x = 2$.

(23) The line $2y - x = 4$ has slope $\frac{1}{2}$ so the equation of a line through the point $(1, 1)$ which is perpendicular to this line is $y - 1 = -2(x - 1)$. The intersection of these lines may be found by solving the latter for $y = -2x + 3$ and substituting into the equation of the first line: $2(-2x + 3) - x = 4$ so $x = \frac{2}{5}$ and $y = \frac{11}{5}$. By Pythagoras, this is the closest point on the line $2y - x = 4$ to the point $(1, 1)$ because the distance to any other point is the hypotenuse of a right triangle with one side being the segment between these points. This distance is $\sqrt{(\frac{2}{5} - 1)^2 + (\frac{11}{5} - 1)^2} = \frac{3\sqrt{5}}{5}$.

(24) The line $y = 2x - 3$ has slope 2 so the equation of a line through the point $(0, 1)$ which is perpendicular to this line is $y - 1 = -\frac{1}{2}(x - 0)$. The intersection of these lines may be found by substituting this into the equation of the first line: $-\frac{1}{2}x + 1 = 2x - 3$ so $x = \frac{8}{5}$ and $y = \frac{1}{5}$. The distance from $(0, 1)$ to this point, hence to the line, is This distance is $\sqrt{(\frac{8}{5} - 0)^2 + (\frac{1}{5} - 1)^2} = \frac{4\sqrt{5}}{5}$.

(25) The point $(0, 0)$ is on the line $y = 2x$. Both lines have slope 2 so the equation of a line through the point $(0, 0)$ which is perpendicular to the line $y = 2x + 3$ line is $y - 0 = -\frac{1}{2}(x - 0)$. The intersection of those lines may be found by substituting one into other: $-\frac{1}{2}x = 2x + 3$ so $x = -\frac{6}{5}$ and $y = \frac{3}{5}$. The distance from $(0, 0)$ to this point,

which is the shortest distance between point on one line and any point on the other, is $\sqrt{(-\frac{6}{5} - 0)^2 + (\frac{3}{5} - 1)^2} = \frac{3\sqrt{5}}{5}$.