#6 Change $dx \to dt$

change $\cos^2 x \to \cos^2 t$

MATH 1210-90 Fall 2011

Final Exam

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Hint: do NOT calculate any numerical value, unless specified otherwise.

LAST NAME ______________________

FIRST NAME Grader's Copy

ID NO. ______________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	30	______
PROBLEM 2	30	______
PROBLEM 3	30	______
PROBLEM 4	30	______
PROBLEM 5	30	______
PROBLEM 6	30	______
PROBLEM 7	30	______
PROBLEM 8	10	______
TOTAL	160	______

PROBLEM 1

(30 pt) Evaluate the limit

$$\lim_{h\to 0} \frac{3(5+h)^2 + 3(5+h) - (3\cdot 5^2 + 3\cdot 5)}{h}$$

method (1) $\lim_{h \to 0} \left[3(5^2 + 10h + h^2) + 3(5+h) - (3 \cdot 5^2 + 3 \cdot 5) \right] / h$

(10pt)

$= \lim_{h \to 0} (30h + 3h^2 + 3h)/h$

(10 pt)

$= \lim_{h \to 0} 30 + 3h + 3 = 33$

(10 pt)

(10 pt)

method (2) $\lim = f'(5)$ (10 pt)

$f(x) = 3x^2 + 3x$

$f'(x) = 6x + 3$ (10 pt)

$= 33$

PROBLEM 2

(30 pt) Find the equation of the tangent line to the curve $y = (x+1)(x^2-1)$ at the point $(1, 0)$.

[check : 0 = LHS

RHS = 2 · 0 = 0]

$y = x^3 + x^2 - x - 1$ or use product rule

$y' = 3x^2 + 2x - 1$ (10 pt)

$m = 3 + 2 - 1 = 4$ (10 pt)

equation : $y = m(x - 1)$

$= 4x - 4$

(10 pt

PROBLEM 4

(30 pt) Consider the parametric equation

$$x = \cos\theta + \theta \sin\theta \qquad y = \sin\theta - \theta\cos\theta$$

What is the length of the curve for $\theta = 0$ to $\theta = 7/2\pi$?

$$\frac{dy}{d\theta} = \cancel{\theta} \cos\theta - \cos\theta + \theta \sin\theta$$

$$= \theta \sin\theta$$

$$\frac{dx}{d\theta} = -\sin\theta + \sin\theta + \theta\cos\theta$$

$$\int_{\theta=0}^{\frac{7\pi}{2}} \sqrt{\left(\frac{dy}{d\theta}\right)^2 + \left(\frac{dx}{d\theta}\right)^2} \, d\theta$$

$$= \int_{\theta=0}^{\frac{7}{2}\pi} \theta \, d\theta$$

$$= \frac{1}{2}\theta^2 \Big|_0^{\frac{7}{2}\pi} \quad \left[= \frac{1}{2} \cdot \frac{49}{4} \cdot \pi^2 \right]$$

PROBLEM 5

(30 pt) Set up the finite integral for the volume formed by rotating the region inside the first quadrant enclosed by

$$y = x^4 \qquad y = 8x$$

(a) about the x-axis.
(b) about the y-axis.
Please do not evaluate the values.

$y = x^4 = 8x$

$x = 0$ or $x = 2$

$y = 0$ $\quad$ $y = 16$ (10pt)

(a) (10pt) $\int_{x=0}^{2} \pi (8x)^2 - \pi (x^4)^2 \, dx$

$\left[= \int_{y=0}^{16} 2\pi y \left(y^{\frac{1}{4}} - \frac{1}{8} y \right) dy \right]$

(10 p)t
(b) $\int_{x=0}^{2} 2\pi x \, (8x - x^4) \, dx$

$\left[= \int_{y=0}^{16} \pi \, (y^{\frac{1}{4}})^2 - (\frac{1}{8} y)^2 \, dy \right]$

$dx \to dt$

$\cos^2 x \to \cos^2 t$

t

PROBLEM 6

(30 pt) Given

$$f(x) = \int_0^x \frac{t^2 - 1}{1 + \cos^2 x}\, dx$$

At what value does the local **max** of $f(x)$ occur?
Hint: use the first derivative test .

$$f'(x) = \frac{x^2 - 1}{1 + \cos^2 x}$$

(Fundamental Thm)

$f'(x) = 0 \qquad x = \pm 1$

f' > 0 < 0 > 0

-1 $+$

f' Inc   Dec   Inc

~~Dec~~

-1 $+$

max   min

$$f(-1) = -\int_{-1}^{0} \frac{t^2 + 1}{1 + \cos^2 t}\, dt$$

PROBLEM 7

(30 pt)

$$\lim_{n\to\infty} \frac{1}{(1+\frac{6}{n})^2}\cdot\frac{6}{n} + \frac{1}{(1+\frac{12}{n})^2}\cdot\frac{6}{n} + \frac{1}{(1+\frac{18}{n})^2}\cdot\frac{6}{n} + \cdots + \frac{1}{(1+\frac{6n}{n})^2}\cdot\frac{6}{n}$$
$$= \int_1^b f(x)\,dx.$$

Find out the upper limit b, and the integrand $f(x)$, and the finite integral $\int_1^b f(x)\,dx$.

$$\frac{6}{n} = \frac{b-a}{n}$$

$$f(x_i) = f\left(i\frac{(b-a)}{n} + a\right)$$

(10pt)

$$a = 1 \Rightarrow b = 7$$

$$f(x_i) = f\left(1 + \frac{6}{n}\cdot i\right)$$

$$f(x) = \frac{1}{x^2} = x^{-2}$$

10pt

$$\int_1^7 f(x)\,dx = -\frac{1}{x}\Big|_1^7$$

(10pt)

$$\left[= 1 - \frac{1}{7}\right]$$

PROBLEM 8

(10 pt) Evaluate the definite integral

$$\int_3^5 \left(\frac{d}{dt}\sqrt{4+3t^4}\right)\,dt$$

$t = 3 \qquad \sqrt{4 + 3t^4} = \sqrt{3^5 + 4}$

$t = 5 \qquad \sqrt{4 + 3 \cdot 5^4} = S_1$

upper bound

Integral

$$= \int_{4}^{\sqrt{\cancel{8}+3\cdot 5^{4}}} 1\,ds \qquad (5pt)$$

$S = \sqrt{4 + 3\,\cancel{5}^{4}}$ 3

$$= S \,\Big|_{\sqrt{4+3^4}}^{\sqrt{\cancel{8}+3\cdot 5^4}} \quad (5pt)$$