

6

change $dx \rightarrow dt$

change $\cos^2 x \rightarrow \cos^2 t$

MATH 1210-90 Fall 2011

Final Exam

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Hint: do NOT calculate any numerical value, unless specified otherwise.

LAST NAME _____

FIRST NAME Grader's Copy

ID NO. _____

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1 30 _____

PROBLEM 2 30 _____

PROBLEM 3 30 _____

PROBLEM 4 30 _____

PROBLEM 5 30 _____

PROBLEM 6 30 _____

PROBLEM 7 30 _____

PROBLEM 8 10 _____

TOTAL 160 _____

PROBLEM 1

(30 pt) Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{3(5+h)^2 + 3(5+h) - (3 \cdot 5^2 + 3 \cdot 5)}{h}$$

method (1) $\lim_{h \rightarrow 0} [3(5^2 + 10h + h^2) + 3(5+h)$

(10 pt) $- (3 \cdot 5^2 + 3 \cdot 5)] / h$

$= \lim_{h \rightarrow 0} (30h + 3h^2 + 3h) / h$

(10 pt)

$= \lim_{h \rightarrow 0} 30 + 3h + 3 = 33$

(10 pt)

(10 pt)

method (2) $\lim = f'(5)$

(10 pt)

$= 33$

(10 pt)

$f(x) = 3x^2 + 3x$

$f'(x) = 6x + 3$

(10 pt)

10

PROBLEM 2

(30 pt) Find the equation of the tangent line to the curve $y = (x+1)(x^2-1)$ at the point $(1, 0)$.

$$[\text{check} : 0 = \text{LHS}]$$

$$\text{RHS} = 2 \cdot 0 = 0]$$

$$y = x^3 + x^2 - x - 1 \quad \text{or use product rule}$$

$$y' = 3x^2 + 2x - 1 \quad (10 \text{ pt})$$

$$m = 3 + 2 - 1 = 4 \quad (10 \text{ pt})$$

$$\text{equation: } y = m(x-1)$$

$$= 4x - 4$$

$$(10 \text{ pt})$$

PROBLEM 4

(30 pt) Consider the parametric equation

$$x = \cos \theta + \theta \sin \theta \quad y = \sin \theta - \theta \cos \theta$$

What is the length of the curve for $\theta = 0$ to $\theta = 7/2\pi$?

$$\begin{aligned} \frac{dy}{d\theta} &= \cancel{0} \cos \theta - \cos \theta + \theta \sin \theta \\ &= \theta \sin \theta \end{aligned}$$

$$\frac{dx}{d\theta} = -\sin \theta + \sin \theta + \theta \cos \theta$$

$$\begin{aligned} &\int_{\theta=0}^{\frac{7\pi}{2}} \sqrt{\left(\frac{dy}{d\theta}\right)^2 + \left(\frac{dx}{d\theta}\right)^2} d\theta \\ &= \int_{\theta=0}^{\frac{7\pi}{2}} \theta d\theta \\ &= \frac{1}{2} \theta^2 \Big|_0^{\frac{7\pi}{2}} \quad \left[= \frac{1}{2} \cdot \frac{49}{4} \cdot \pi^2 \right] \end{aligned}$$

PROBLEM 5

(30 pt) Set up the finite integral for the volume formed by rotating the region inside the first quadrant enclosed by

$$y = x^4 \quad y = 8x$$

(a) about the x -axis.

(b) about the y -axis.

Please do not evaluate the values.



$$y = x^4 = 8x$$

$$x = 0 \quad \text{or} \quad x = 2$$

$$y = 0 \quad y = 16 \quad (10 \text{ pt})$$

$$(10 \text{ pt}) \quad (a) \quad \int_{x=0}^2 \pi (8x)^2 - \pi (x^4)^2 dx$$

$$[= \int_{y=0}^{16} 2\pi y (y^{\frac{1}{4}} - \frac{1}{8}y) dy]$$

$$(10 \text{ pt}) \quad (b) \quad \int_{x=0}^2 2\pi x (8x - x^4) dx$$

$$[= \int_{y=0}^{16} \pi (y^{\frac{1}{4}})^2 - (\frac{1}{8}y)^2 dy]$$

$$dx \rightarrow dt$$

$$\cos^2 x \rightarrow \cos^2 t$$

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PROBLEM 6

(30 pt) Given

$$f(x) = \int_0^x \frac{t^2 - 1}{1 + \cos^2 t} dt$$

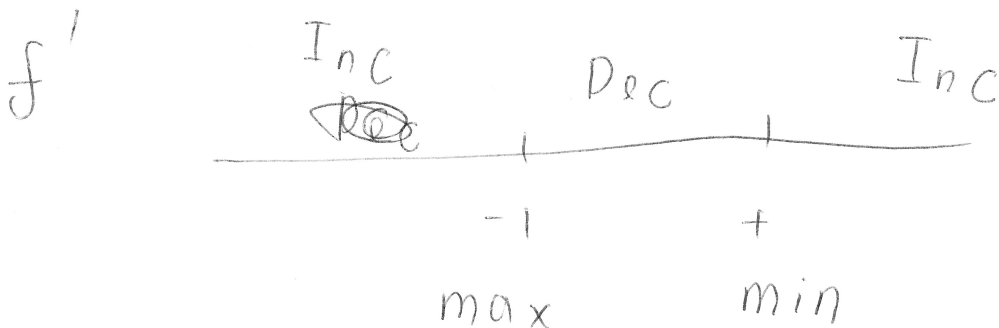
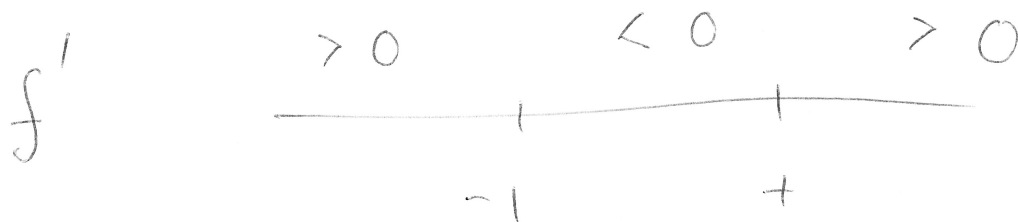
At what value does the local **max** of $f(x)$ occur?

Hint: use the first derivative test.

$$f'(x) = \frac{x^2 - 1}{1 + \cos^2 x}$$

(Fundamental Thm)

$$f'(x) = 0 \quad x = \pm 1$$



$$f(-1) = -\int_{-1}^0 \frac{t^2 + 1}{1 + \cos^2 t} dt$$

PROBLEM 7

(30 pt)

$$\lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{6}{n})^2} \cdot \frac{6}{n} + \frac{1}{(1 + \frac{12}{n})^2} \cdot \frac{6}{n} + \frac{1}{(1 + \frac{18}{n})^2} \cdot \frac{6}{n} + \cdots + \frac{1}{(1 + \frac{6n}{n})^2} \cdot \frac{6}{n}$$

$$= \int_1^b f(x) dx.$$

Find out the upper limit b , and the integrand $f(x)$, and the finite integral $\int_1^b f(x) dx$.

$$\frac{6}{n} = \frac{b-a}{n}$$

$$f(x_i) = f\left(i \cdot \frac{b-a}{n} + a\right)$$

(10 pt) $a=1 \Rightarrow b=7$

$$f(x_i) = f\left(1 + \frac{6}{n} \cdot i\right)$$

$$f(x) = \frac{1}{x^2} = x^{-2}$$

$$\int_1^7 f(x) dx = -\frac{1}{x} \Big|_1^7$$

(10 pt)

$$\left[= 1 - \frac{1}{7} \right]$$

10pt

PROBLEM 8

(10 pt) Evaluate the definite integral

$$\int_3^5 \left(\frac{d}{dt} \sqrt{4+3t^4} \right) dt$$

$$t = 3 \quad \sqrt{4+3t^4} = \sqrt{3^4+4}$$

$$t = 5 \quad \sqrt{4+3 \cdot 5^4} = S_1$$

upper bound

Integral

$$= \int \frac{\sqrt{4+3t^4}}{4} dt \quad (5 \text{ pt})$$

$$S = \int_3^5 \sqrt{4+3t^4} dt$$

$$= S \left| \frac{\sqrt{4+3t^4}}{4} \right|_3^5 \quad (5 \text{ pt})$$