Calculus I
Practice Final Exam

Do as many problems as you can in two hours. (Then do them all).

1. Find the derivatives of the following functions:

a) $f(x) = (x^3 - 1)(x^2 + 1)^2$

b) $g(x) = \dfrac{\sin x}{\cos x + 1}$

2. Find the derivatives of the following functions:

a) $f(x) = \sin^3(4x + 1)$

b) $g(x) = \int_1^x (1 + t^2)t\,dt$

3. Integrate:
a) $\int (x^2 + 1)^2 x\,dx$

b) $\int \tan x \sec^2 x\,dx$

4. Integrate:

a) $\int_1^4 \dfrac{1}{\sqrt{y}(\sqrt{y} + 1)^2}\,dy$
b) $\int_0^{\pi/2} \cos^2 x \sin x\,dx$

5. Find the slope of the tangent line to the curve $\cos x + \sin y = 3/2$ at the point $(\pi/3, \pi/2)$.

6. A conical water tank of height 8 ft, base radius 5 ft, stands on its vertex. Water is flowing in at the top at a rate of 2.5 ft^3/min. At what rate is the water level rising when that level is at 3 ft? The volume of a cone of base radius r and height h is $(1/3)\pi r^2 h$.

7. A farmer wishes to enclose a rectangular field of 10,000 square yards so that one side is brick and the other three sides are chain link fence. A Brick wall costs $18 a linear yard and chain link, $ 6 a linear yard. Find the dimensions of the field which minimizes the cost.

8. Find the solution to the differential equation

$$\frac{dy}{dx} = y^2x^2 + y^2$$

such that $y(1) = 2$.

9. Graph

$$y = \frac{x^3}{x^2 - 1}$$

showing clearly all asymptotes and local maxima and minima.

10. What is the area of the region in the right half plane bounded by the curves $y = x^3 - 3x$ and $y = 3x$.

11. The region in the first quadrant under the curve $y^2 = 2x - x^2$ is rotated about the x-axis. Find the volume of the resulting solid.

12. The region between the curves $y = 8x$ and $y = x^4$ is rotated about the y-axis. Find the volume of the resulting solid.

13. Find the length of the curve $y = t^3$, $x = t^2$, $0 \leq t \leq 1$.

14. Find the work done in pumping all the oil (whose density is 50 lbs. per cubic foot) over the edge of a cylindrical tank which stands on end. Assume that the radius of the base is 4 feet, the height is 10 feet and the tank is full of oil.

15. Find the center of mass of the homogeneous region in the first quadrant bounded by the curve $x^4 + y = 1$.