

Calculus I
Practice Exam 3, Summer 2002

1. Find the indefinite integral of the given function: a) $f(x) = x^2 - 3x + x^{-2}$ b) $g(x) = \sin x + \frac{1}{\cos^2 x}$

2. Find the function whose value at 0 is 0 and whose derivative is

a) $\frac{x}{(2x^2 + 1)^2}$

b) $\frac{\sin x}{\cos^4 x}$

c) $\frac{\sin^2 x}{\cos^4 x}$

3. Find y as a function of x , given that $y = 4$ when $x = 0$ and

$$\frac{dy}{dx} = x + \sin x .$$

4. Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{x}{y^2}$$

such that $y(1) = 2$.

5. Calculate the definite integrals:

a) $\int_{-4}^4 (x^3 + 3x + \sin(2x)) dx$

b) $\int_0^{\pi/2} (\sin x \cos x) dx$

6. Find the definite integrals:

a) $\int_1^3 x(x+1)^2 dx =$

b) $\int_0^{\pi} (\sin x + \cos x) dx$

7. Find the area of the region bounded by the curves $y = x^3 - x^2 + x$ and $y = x^3 + 2x^2 - 10$.

8. Find the area of the region bounded by the curves $y = x^3$ and $y = x^2 + 2x$.

9. A solid lies above the region in the first quadrant bounded by the curve $y = \sec x$ from $x = 0$ to $x = \pi/4$, so that a cross-section above each line $x = \text{constant}$ is a square. What is the volume of the region?

10. The region in the first quadrant bounded by the curves $y = x - 1$ and $y = 3 - x$ is rotated about the x -axis. What is the volume of the resulting solid? What is the answer if the region is rotated about the y -axis?