

**Calculus III**  
**Exam 2, Summer 2003**

**You may use graphing calculators. Each problem is worth 20 points. You MUST show your work. Just the correct answer is not sufficient for any points.**

1. The relation

$$\sqrt{y} + xy^2 = 34$$

determines a curve in the  $x$ - $y$  plane. Find the slope of the line tangent to the curve at the point  $(2,4)$ .

2. Let

$$y = \frac{x}{x^2 + 1}.$$

- a) Find the  $x$  coordinate of the points of local maxima and minima of  $y$ .
- b) Find the  $x$  coordinate of the points of inflection of the curve so defined.

3. A monument to the mathematicians of Utah is to be built in a cylindrical form so as to enclose 6000 cu. ft. The bottom and top are to be faced with copper, costing \$30 a sq. ft., and the side surfaces of the cylinder are to be faced with stone, costing \$12 a sq. ft. What should the radius of the base of the cylinder be so as to minimize the total cost of the facing? Recall: A cylinder of base radius  $r$  and height  $h$  has volume  $\pi r^2 h$ , the area of the cylindrical side is  $2\pi r h$ , and the areas of the base and top are each  $\pi r^2$ .

4. Let  $y = x^6 - 3x^4$ . Find the intervals in which the function is increasing and decreasing, and where it is concave up and concave down.

5. Graph the function

$$y = x^2 + \frac{1}{x^2}$$

showing clearly all asymptotes and local maxima and minima.