Calculus I
Exam 1, Summer 2003, Answers

1. Find the equation of the line which goes through the point (1,2) and is parallel to the line given by the equation $3x+y=1$.

Answer. If we write the equation as $y=1-3x$, we see that the slope is -3. $(1,2)$ is on the line, so we use the point-slope form:

$$\frac{y-2}{x-1}=-3$$

Simplifying, we get $y-2=-3(x-1)=-3x+3$ or $y=-3x+5$.

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2. Find the derivatives of the following functions:

a) $f(x)=x^2+1$

Answer. $f'(x)=2x$.

b) $f(x)=x+\dfrac{1}{x}$

Answer. Rewrite this as $f(x)=x+x^{-1}$. Then $f'(x)=1-x^{-2}$.

c) $f(x)=(x+x^{-1})(x^2+1)$;

Answer. We use the product rule

$$f'(x)=(x+x^{-1})(2x)+(x^2+1)(1-x^{-2})$$

and then simplify:

$$f'(x)=2x^2+2+x^2-1+1-x^{-2}\text{ , or }\quad f'(x)=3x^2+2-x^{-2}$$

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3. Find the derivatives of the following functions:

a) $f(x)=\dfrac{x}{x^2+1}$

Answer. Use the quotient rule:

$$f'(x)=\frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$$

b) $f(x)=\dfrac{1+\tan x}{1-\tan x}$

Answer. Use the addition formula for the tangent: $f(x)=\tan(x+\pi/4)$. Then differentiate: $f'(x)=\sec^2(x+\pi/4)$. If you used the quotient rule, you probably ended up with

$$f'(x)=\frac{(1-\tan x)(\sec^2 x)-(1+\tan x)(-\sec^2 x)}{(1-\tan x)^2}=\frac{2\sec^2 x}{(1-\tan x)^2}$$

which is also correct.

4. At what points (x, y) does the graph of the function $y = x^2 - x^3$ have horizontal tangent line (a line with slope 0)?

Answer. Taking the derivative we have $dy/dx = 2x - 3x^2$. This gives the slope of the tangent line at the general point, so we are looking for the values of x where this is zero. Solve $2x - 3x^2 = 0$ to get $x = 0$ and $x = 2/3$. Now solve for the corresponding values of y, finding the points $(0, 0)$ and $(3/2, 4/27)$.

5. Let C be the curve given by the equation $y = (2x+1)^3 - 12x^3$. Find the equation of the tangent line to C at the point (2, 29).

Answer. To get the slope of the tangent line, differentiate: $dy/dx = 3(2x+1)^2(2) - 36x^2$. Now, at $x = 2$, we have the slope $m = 3(5)^2(2) - 36(2^2) = 6$. Thus, in point-slope form, the equation is

$$\frac{y-29}{x-2} = 6 \text{ , which simplifies to } \quad y = 6x + 17 \text{ .}$$