1210-90 Exam 1

Summer 2013

Name KEY

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answers.

1. (24pts) Compute the following limits. Be sure to show your work. **Note:** Answers can be values, $+\infty$, $-\infty$, or DNE (does not exist). An answer of DNE requires some explanation!

(a) $\lim_{x\to 1} \frac{1}{x^2-5}$ $= \frac{1}{1^2-5} = -\frac{1}{4}$ Missing limits -3

4

(b) $\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$ $= \lim_{x\to 2} \frac{(x-2)(x-1)}{x-2} = \lim_{x\to 2} (x-1) = 1$

4

(c) $\lim_{x\to -\infty} \frac{3x^2-2}{x^3+5x}$ $= \lim_{x\to -\infty} \frac{x^2(3-2/x^2)}{x^3(1+5/x^2)} = \left(\lim_{x\to -\infty} \frac{1}{x}\right)\left(\lim_{x\to -\infty} \frac{3-2/x^2}{1+5/x^2}\right)$

$= 0 \cdot 3 = 0$

4

(d) $\lim_{x\to +\infty} \frac{7x^4+9}{x^4-x^3}$ $= \lim_{x\to \infty} \frac{x^4(7+9/x^4)}{x^4(1-1/x)} = \lim_{x\to \infty} \frac{7+9/x^4}{1-1/x} = 7$

4

(e) $\lim_{x\to 0} \frac{x^2-1}{x}$ DNE +2 no justification

$\lim_{x\to 0^+} \frac{x^2-1}{x} = -\infty$ while $\lim_{x\to 0^-} \frac{x^2-1}{x} = +\infty$

4

(f) $\lim_{x\to 1^-} \frac{1}{x^2-1}$ $= \lim_{x\to 1^-} \frac{1}{(x+1)(x-1)} = \left(\lim_{x\to 1^-} \frac{1}{x+1}\right)\left(\lim_{x\to 1^-} \frac{1}{x-1}\right)$

DNE +1

$= \frac{1}{2}(-\infty) = -\infty$

4

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2. (24 pts) Compute the following derivatives. There is no need to simplify.

(a) $D_x(5x^3 - 2x^2 + 1)$

4 $= 15x^2 - 4x$

(b) $D_x(\frac{x^2+2}{x})$

4 $= \frac{x(2x) - (x^2+2)(1)}{x^2} = \frac{2x^2 - x^2 - 2}{x^2} = \frac{x^2-2}{x^2}$

(c) $D_x(\sin(x)\cos(x))$

4 $= \sin x(-\sin x) + \cos(x)\cos(x)$

$= \cos^2 x - \sin^2 x$

(d) $D_x((x^2 - x)^7)$

4 $= 7(x^2-x)^6(2x-1)$

(e) $D_x(x^2 \sin(x^2))$

4 $= 2x\sin(x^2) + x^2\cos(x^2)(2x)$

$= 2x\sin(x^2) + 2x^3\cos(x^2)$

(f) $D_x\left((\frac{\tan x}{x})^5\right)$

4 $= 5\left(\frac{\tan x}{x}\right)^4\left(\frac{x\cdot\sec^2 x - \tan x}{x^2}\right)$

24

3. (10pts) Compute the derivative of $f(x) = 5x^2 + 3$ by using **the definition of the derivative**; that is, compute the limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

5

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{5(x+h)^2+3-(5x^2+3)}{h}$$

$$= \lim_{h\to 0} \frac{5x^2+10xh+5h^2+3-5x^2-3}{h}$$

Missing limits −3

$$= \lim_{h\to 0} \frac{10xh+5h^2}{h}$$

$$= \lim_{h\to 0} (10x+5h) = 10x$$

10

4. (10pts) Find the equation of the tangent line to the graph of the function $f(x) = 4x^5 - 3x^3 + 2$ at the point $(-1, 1)$.

$$f'(x) = 20x^4 - 9x^2$$

$$\Rightarrow f'(-1) = 11.$$

$$f(-1) = 1$$

$$y = f(-1) + f'(-1)(x+1)$$

$$y = 1 + 11(x+1) = 11x + 12$$

10

5. (4pts) Find $\frac{d^2y}{dx^2}$ if $y = \sin(2x)$.

$$\frac{dy}{dx} = 2\cos(2x)$$

$$\frac{d^2y}{dx^2} = -4\sin(2x)$$

4

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6. (10pts) An object thrown upwards at time $t = 0$ (measured in seconds) has a height (measured in feet) at time t given by $h(t) = -16t^2 + 96t$.

(a) (3pts) Find the velocity of the object at time t.

3

$v(t) = h'(t) = -32t + 96$

(b) (4pts) At what time does the object reach its maximum height and what is its maximum height?

4

$0 = v(t) = -32t + 96 \Rightarrow t = 3$ 2

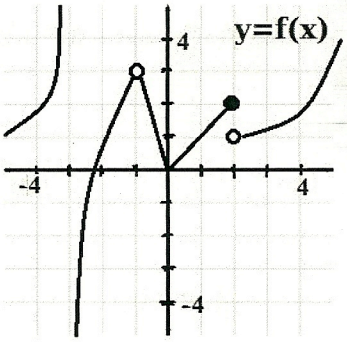
$h(3) = -16(3)^2 + 96(3) = 144$ ft 2

(c) (3pts) What is the acceleration of the object at time t?

3

$a(t) = h''(t) = -32$

7. (12pts) Examine the provided graph of the function $f(x)$ below and fill in the blank provided.



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(a) $\lim_{x \to 2^+} f(x) =$ 1

(b) $\lim_{x \to 2^+} f(x) =$ 1

→ I intended the question to be $\lim_{x \to 2^-} f(x) = 2$

(c) F True (T) or False (F): f is continuous at $x = 2$.

(d) F True (T) or False (F): f is continuous at $x = -1$.

(e) $x =$ -3 is a vertical asymptote.

(f) F True (T) or False (F): f is differentiable at $x = 0$.

8. (6pts) Give a function that satisfies the stated criteria. **Note:** There are many possible answers for each blank!

3 (a) (3pts) A function that is not continuous at $x = 0$. $y =$ $1/x$

3 (b) (3pts) A function whose derivative does not exist at $x = 0$. $y =$ $|x|$

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