1210-90 Exam 1
Fall 2013

Name _____

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answers.

1. (24pts) Compute the following limits. Be sure to show your work. **Note:** Answers can be values, $+\infty$, $-\infty$, or DNE (does not exist). An answer of DNE requires some explanation!

   (a) $\lim_{x\to 0} \sqrt{x^2+4}$

   (b) $\lim_{x\to 3} \dfrac{x^2-7x+12}{x-3}$

   (c) $\lim_{x\to 0} \dfrac{\sin(2x)}{x}$

   (d) $\lim_{x\to 1} \dfrac{|x-1|}{x-1}$

   (e) $\lim_{x\to +\infty} \dfrac{4x^2+9}{x^2-x}$

   (f) $\lim_{x\to 0} \dfrac{x^3-4}{x^2}$

2. (12pts) Suppose c is a constant and consider the piecewise-defined function

$$f(x) = \begin{cases} x^2 - 4x + c^2, & x < 1 \\ -3cx + 1, & x \geq 1 \end{cases}$$

(a) (4pts) Compute $\lim_{x \to 1^-} f(x)$

(b) (4pts) Compute $\lim_{x \to 1^+} f(x)$

(c) (4pts) Find the value(s) of c that make $f(x)$ continuous at $x = 1$.

3. (10pts) Use **the definition of the derivative** to compute $f'(1)$ for $f(x) = x^3 + x$; that is, compute the limit

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

4. (20 pts) Compute the following derivatives. There is no need to simplify.

   (a) $D_x(x^7 - 5x^5 + 3x)$

   (b) $D_x((x^3 + x)\sin x)$

   (c) $D_x(\frac{x^3}{x^2+1})$

   (d) $D_x(\cos(7x^2 + 9))$

   (e) $D_x(\left(\frac{\sin x}{x}\right)^5)$

5. (6pts) Find $\frac{d^3y}{dx^3}$ if $y = 5x^3 + x - \sin x$.

6. (8pts) Find the equation to the tangent line to the graph of the function $f(x) = \frac{1}{(x-2)^2}$ at the point $(1, 1)$.

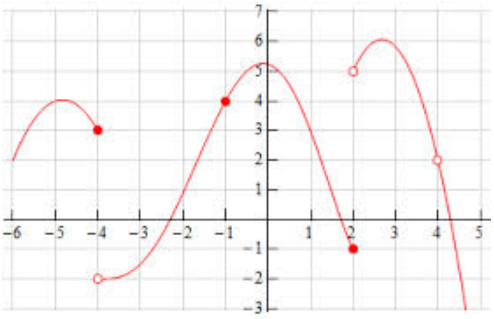
7. (8pts) Let

$$f(x) = \left(\frac{x}{1+x^2}\right)^7$$

(a) (4pts) Find the derivative $f'(x)$.

(b) (4pts) At what three points x is the tangent line to the graph of $y = f(x)$ horizontal?

8. (12pts) Examine the graph of the function $f(x)$ below and fill in the blanks.



(a) $\lim_{x\to -4^-} f(x) =$ _____

(b) $\lim_{x\to -4^+} f(x) =$ _____

(c) List all values of x, $-6 < x < 5$, where $f(x)$ is **not** continuous. $x =$_____

(d) _____ True (T) or False (F): $f'(1) > 0$.

(e) _____ True (T) or False (F): $f'(-2) > f'(-3)$

(f) _____ True (T) or False (F): f is differentiable at $x = 4$.