

MATH CIRCLE FINALE

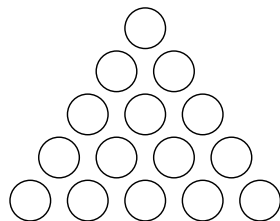
May 1, 2002

1. There are four of you who wish to cross a bridge at night. Alas, to your dismay, you only have one torch. You cannot go across the bridge without the torch, but since the torch is dim only two people can cross the bridge at the same time. The pair may only travel as fast as the slowest link. The people in your group can traverse the bridge in 1,2,5, and 10 minutes respectively. Prove or disprove: the shortest possible time for all four of you to end up on the other side of the bridge is 19 minutes.

2. A new internet start-up employs 36 employees. The office is located on the first floor of a square building which is 60 feet by 60 feet. Each employee has a 10-by-10 cubicle, and each cubicle has doors to each adjacent cubicle. The cubicles in the northeast and southwest corners of the building also have exits out of the building. The mailman wants to start at one entrance, visit each cubicle exactly once, and leave out the other exit. Can he do it? (Justify your answer.)

3. Suppose you are given 19 cubes whose edges all have length 1. Place one cube centered at the origin. Stack three cubes in the each of the coordinate directions  $(\pm x, \pm y, \pm z)$ . Consider the polyhedron, say  $P$ , formed by the edges of the stacked cubes. Compute the number of vertices minus the number of edges plus the number of faces in  $P$ .

4. Suppose 15 pennies are arranged in a triangle, as in the following diagram:



Show that inside this triangle, there is guaranteed to be an equilateral triangle in which all the vertex pennies are facing the same way (either all heads up or all tails up).

5. Suppose you toss five fair six-sided dice. What is the probability that at least one 1, at least one 2, and at least one 3 appear in the toss?